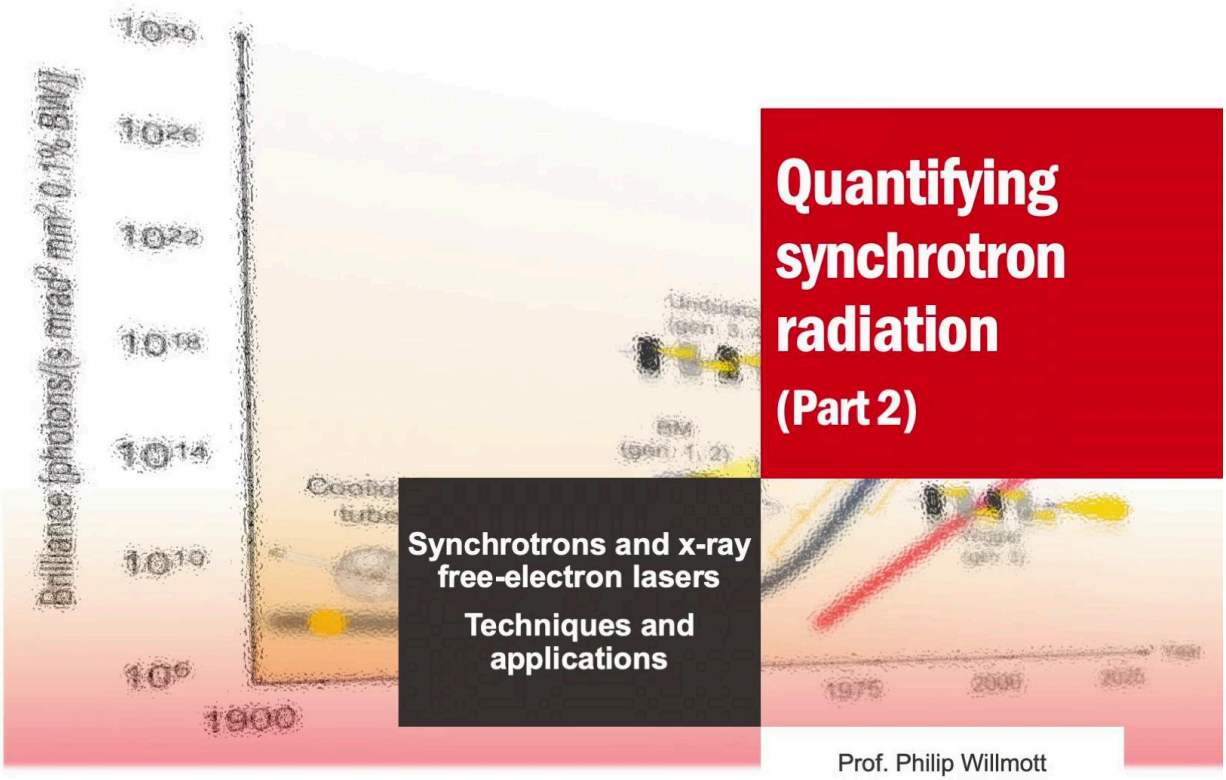
EPFL



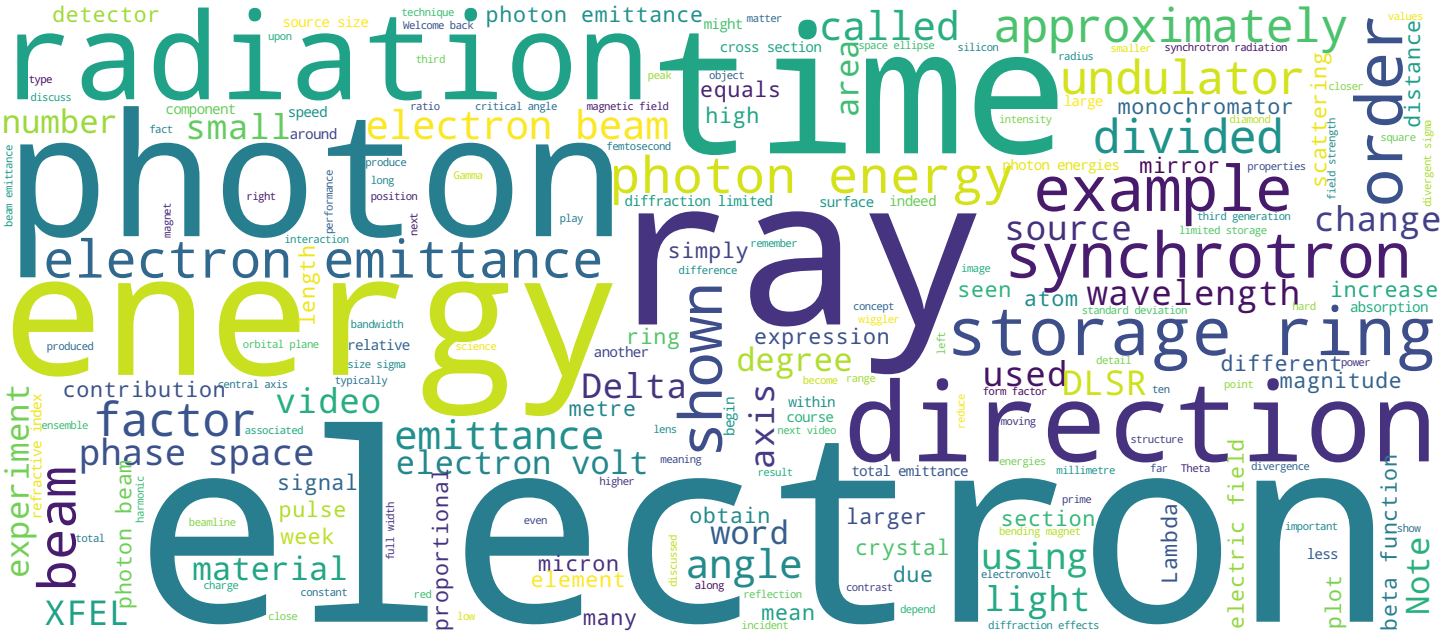
Quantifying synchrotron radiation (Part 2)

Synchrotrons and x-ray free-electron lasers

Techniques and applications

École polytechnique fédérale de Lausanne

Prof. Philip Willmott



EPFL

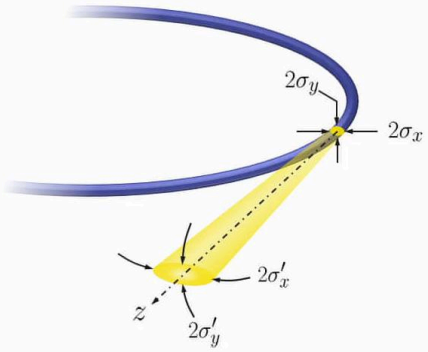
In this video, we'll take a closer look at emittance using the example of an ensemble of electrons. This leads us to the concept of the beta function and how this can be manipulated using optical elements. These ideas underpin so-called phase space matching between the electron beam on the one hand and the photon beam on the other, which in the case of diffraction limited storage rings, is critical in optimising their performance.

Notes

Summary

0m 05s

Electron emittance at synchrotrons



- Divergence and source size of electron beam
  - x-plane
    - Orbital plane
    - More perturbations
    - Larger ε
  - y-plane
    - Less perturbations
    - Smaller ε ~ 5 pm rad (electrons only!!)
- Contribution to ε also from photon emission
  - Discussed shortly

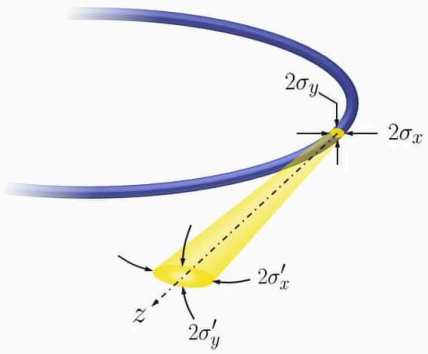
The total emittance is composed of a contribution from the electron beam in the storage ring and the photon beam it emits. We will discover later in this section that the photon beam emittance is caused by diffraction effects and the wave nature of light and depends solely on the photon energy. In contrast, the electron emittance can be minimised by sophisticated magnet design, such as exploited in the latest generation of synchrotrons called diffraction limited storage ring or DLSRs. Before the advent of DLSRs, the contribution from the electrons to the emittance dominated and the photon contribution was negligible. In DLSRs, the electron emittance has been reduced so much that the photon emittance begins to play a significant role. This is the meaning of the term diffraction limited storage ring. The performance of the facility is limited by the fundamental diffraction effects associated with the photon emittance rather than the technical limit set by the electron beam performance. So let's begin by recapping emittance as we've discussed it so far, concentrating for the time being on the electron emittance as it circulates the storage ring.

Notes

Summary

0m 36s

Electron emittance at synchrotrons



- Divergence and source size of electron beam
  - x-plane
    - Orbital plane
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    - Larger ε
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  - Discussed shortly

The emittance in the orbital plane or on the X direction is in general larger than in the vertical Y direction. The main reason for this is that most of the perturbations to and forces acting on the electron beam are in the horizontal orbital plane. Typical values for a third generation facility are about five nanometre radians horizontally and as low as five picometre radians in the vertical plane. Note that this is the electron emittance. In the Y direction, this is significantly smaller than typical values for the photon beam emittance. This means that in this direction, the Y direction, the total emittance is dominated by the photon emittance. In the horizontal direction, however, five nanometre radians is larger than the photon beam emittance for all photon energies above around 20 electron volts, and hence, represents the main contribution in the orbital plane.

Notes

Summary

1m 57s

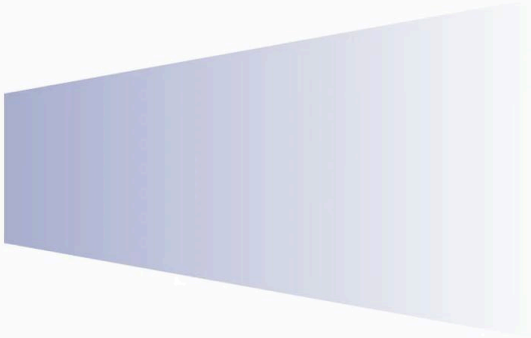
Electron emittance at synchrotrons

3rd generation

$\epsilon_x^e \sim 5000\,\mathrm{pm\,rad}$

$\sigma_x^e \sim 100\,\mu\mathrm{m}$

$\sigma_x'^e \sim 50\,\mu\mathrm{rad}\ (0.00675^\circ\ \mathrm{FWHM})$



FWHM$_x$ @ 13 m ~ 2 mm

4th generation (DLSR)

$\epsilon_x^e \sim 100\,\mathrm{pm\,rad}$

$\sigma_x^e \sim 10\,\mu\mathrm{m}$

$\sigma_x'^e \sim 10\,\mu\mathrm{rad}\ (0.00135^\circ\ \mathrm{FWHM})$

FWHM$_x$ @ 13 m ~ 0.3 mm

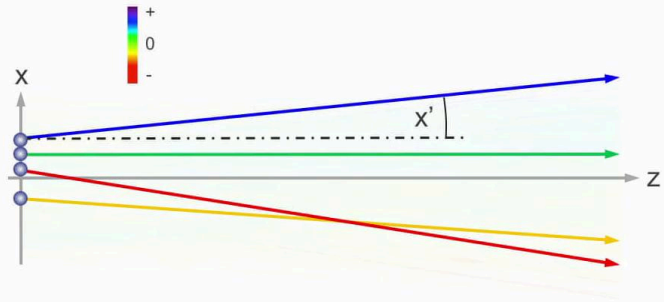
How is this emittance divided up between source size and divergence? This varies along the ring, according to the optics installed. But for a third generation facility typically might consist of a 100 micron standard deviation for the source size or a full width of half, maximum of 236 microns, and the standard deviation divergence of 50 micron radians or in degrees, three millidegrees, equating to a full width half a maximum of approximately seven millidegrees. The contribution to the synchrotron radiation from the electron beam would therefore be two millimetres full width half maximum 13 metres from the source. Now, in the case of a DLSR, the electron emittance might be 50 times smaller to 100 picometre radians with standard deviation source size sigma EX of 10 microns and divergent sigma prime EX, also 10, but now [inaudible 00:04:12]. The electron beam contribution to the size of the radiation is about 0.3 millimetres at 13 metres.

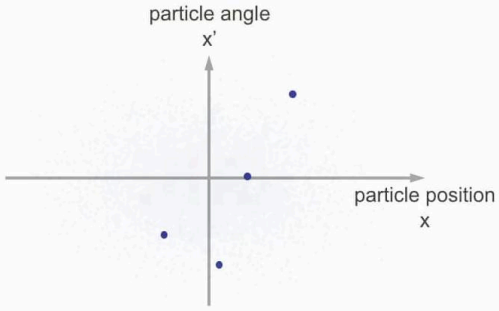
Notes

Summary

3m 03s

Phase space





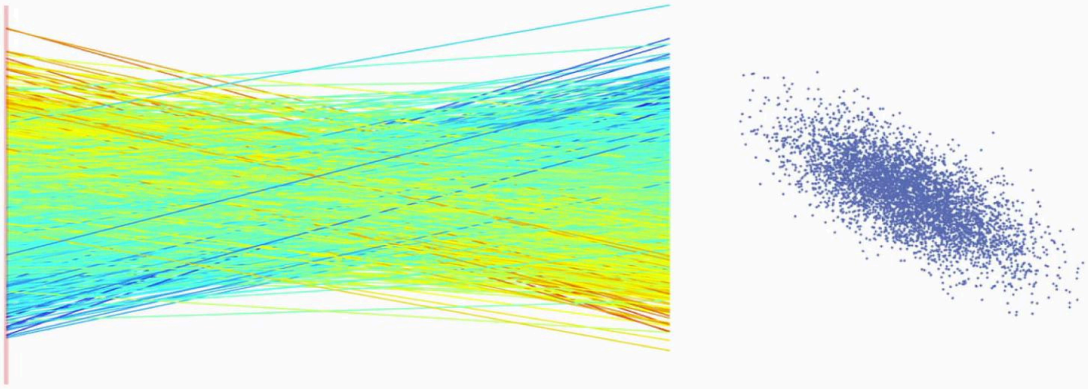
Now let's look at so-called phase space. This is simply just a fancy term describing the distribution of a collection or ensemble of particles, positions and angles within a beam. There is therefore a certain phase space distribution in the X direction or orbital plane and another independent distribution at right angles in the Y direction. We're going to concentrate here on the X direction. So our first electron is seen to be at a certain position away from the central axis of the beam in the X direction and has an angle X prime relative to the same axis. We put this on a graph of X prime versus X. Moving on to the next electron, it seemed to be closer to the central axis and moving with a negative angle. So we add our second data point here on our X versus X prime graph. We continue the third, and the fourth electrons, and so on, and so forth until we have the complete ensemble of electrons in the beam.

Notes

Summary

4m 21s

Phase space



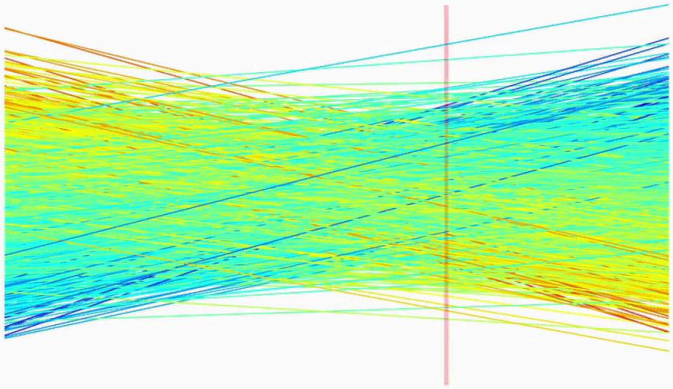
The plot on the right of X versus X prime is the phase space plot of the electrons at the orbital position Z given by the pink semi transparent line. Now, listen, clearly, even though it might look like it, this plot of blue dots is not a 2D cross-section through the beam, only the horizontal axis as units of length. The y-axis here represents angles, not a spatial dimension. The area of the plot has dimensions of angle times distance, and indeed the area of the blob of dots is proportional to the emittance. If we squeeze the electron beam using electron optics into a typed beam, the associated angles increased by the same factor as the extent in that direction decreases. The emittance, therefore, remains constant because the product of the angle and position is unchanged, even though the general elliptical form of the ensemble of dots changes its aspect ratio. But how does the form of the phase space ellipse change as the electrons propagate around the storage ring? Now, unless an electron angle relative to the central axis is just coincidentally zero, its distance from that axis will change as it moves around the ring.

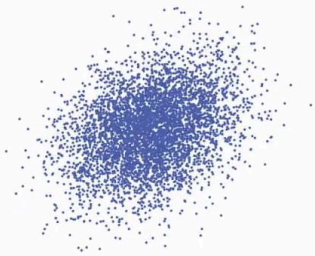
Notes

Summary

5m 39s

Phase space





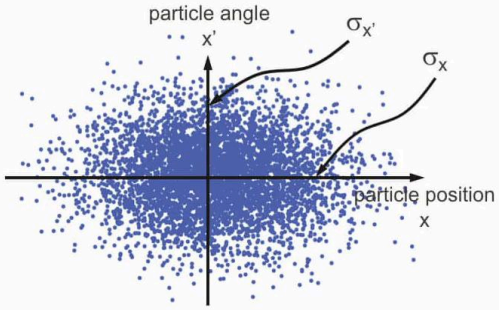
So the dot representing a given electron will keep a constant y-axis value and the angle remains unchanged and assuming no collisions between electrons. But its position in phase space will drift along the x-axis as it propagates forward. We show both the spatial changes and the face based plot here for a beam that converges to a focal minimum and then diverges again on the other side. The current position in phase space is given by the semi transparent pink line. As the beam converges, the phase space ellipse transforms from being skewed top left to bottom right to being symmetric at the focal waist, and then it skews in the other direction as the beam diverges again.

Notes

Summary

7m 07s

Phase space



$$\epsilon_x = \sigma_x \sigma'_x$$

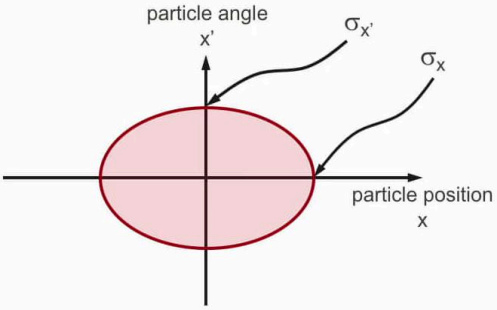
We have seen that the area of the ellipse remains the same, independent of optics and the position along the storage ring. In other words, the emittance is invariant for a given ring. If by using optics, you tighten the beam and reduce its size sigma X by a factor M, you will automatically increase the divergent sigma prime X by the same factor M, meaning the emittance epsilon X doesn't change. We've seen that the area of the ellipse remains the same, independent of optics and the position along the storage ring. In other words, the emittance is invariant for a given ring. If by using optics, you tighten the beam and reduce its physical size sigma X by a factor M, you will automatically increase the divergent sigma X prime by the same factor M, meaning the emittance epsilon X doesn't change.

Notes

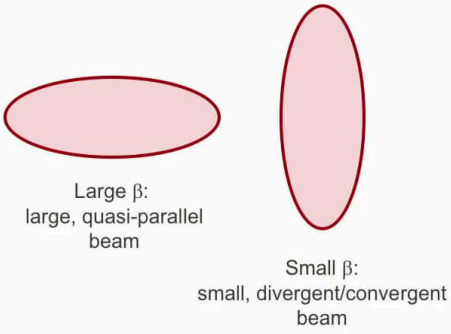
Summary

8m 02s

Phase space



$$\beta_x = \frac{\sigma_x}{\sigma'_x}$$



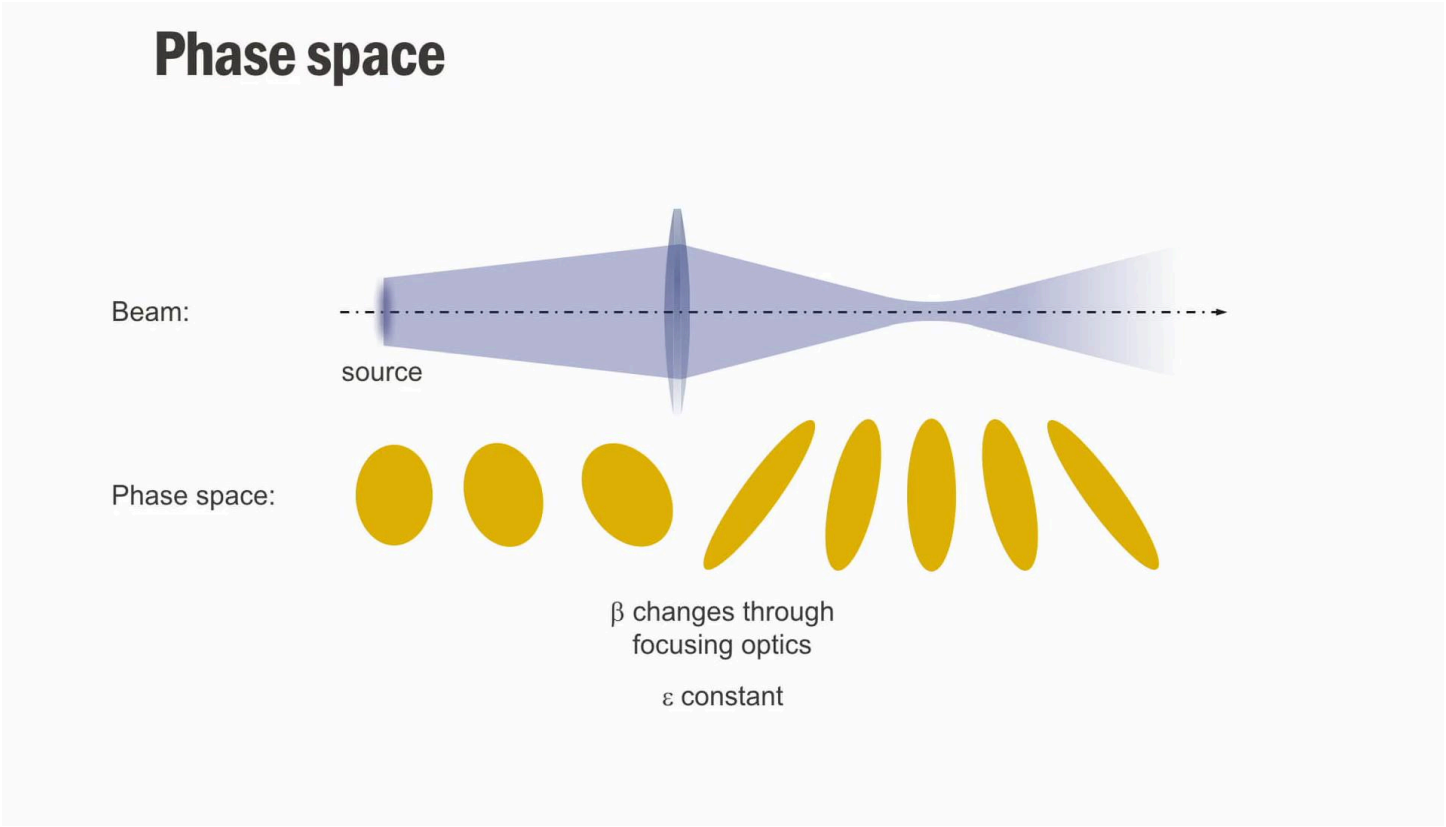
$$\epsilon_x = \sigma_x \sigma'_x$$

In contrast, the ratio of sigma X to sigma X prime called the beta function will change. So, for example, a quasi parallel beam is necessarily larger in spatial extent than a tightly focussed but divergent beam with the same emittance. The former has a large beta function, the latter a small one.

Notes

Summary

9m 05s



I summarise what we have learnt so far in this cartoon of a divergent source of electrons being focussed by electron optics, causing it to go through a narrow focussed waste and subsequently diverge again. Note that the skew of the phase space ellipse changes while there is a sudden change in the beta function due to the optics. Throughout, however, the emittance remains unchanged. In other words, the area of the ellipses are all equal. So for illustrative purposes, we have only considered the emittance of the electron beam. But the photons also have an intrinsic emittance of their own. This emittance is also referred to as the single electron emittance, that is the intrinsic divergence of photons emitted by an electron relative to that electrons trajectory. This is a fundamental property that can't be manipulated and is a consequence of wave optics or diffraction effects. Until the advent of fourth generations DLSRs, the contribution of the photon emittance to the total emittance was insignificant for any beamline operating above a few tens of electron volts. Now, however, it behoves us to consider the photon emittance more carefully and how to best match the electron emittance to it, as this plays a deciding role in DLSR facilities. We'll consider these aspects in the next video.

Notes

Summary

9m 28s